Suction surface:

$B = 0.47078$
$C = -0.12464$
$D = -29.63788$
$E = 10.06706$
$F = 0$

$$x^2 + (0.47078)xy + (-0.12464)y^2 + (-29.63788)x + (10.06706)y = 0$$

Pressure surface:

$B = 0.17614$
$C = -30.39820$
$D = 14.59569$
$E = 13.26829$

$$x^2 + (0.17614)y^2 + (-30.39820)x + (14.59569)y + 13.26829 = 0$$

Once the values for the coefficients have been determined, the curve properties are calculated routinely.

Reference

[1] Deich, M. E., "Flow of gas through turbine latices," English transl., NACA TM 1393, Washington, D. C. (May 1956).

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Design Optimization of Aircraft Structures with Thermal Gradients

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A technique of optimizing a structure subjected to a thermal gradient has been developed. The derivation of optimization equations is demonstrated for three types of basic structure: 1) a honeycomb compression panel, 2) a skin-stringer compression element, and 3) an *I*-beam section. The optimization on these typical structural elements is performed maintaining the interdependency of the structural configuration and the temperature distribution. The procedures also maintain such variables as material properties and a nonlinear stress-strain relation. The complexity of the applied stress and allowable equations has dictated a change in approach of the optimization problem to one of allowable and applied strains rather than allowable and applied stresses. It must be concluded from the results of this optimum design study that it is feasible to account for thermal stress and temperature effects in the preliminary design stages of an aircraft structure.

Nomenclature

A = defined by Eq. (2)
A_S, A_T, A_L, A_F, A_n = area, in.^{2}
a = width of panel, loaded edge, in.
b = length of panel, unloaded edge, in.
b_s = stringer spacing, in.
C = core depth, in.
C_s = effective area coefficient
c_1, c_2, c_3, c_4 = edge fixity constants, Ref. 1
CL = cord length, in.
E = Young's modulus, psi
e = strain, in./in.
F = panel buckling constant
F_{ci} = intercell buckling stress, psi
F_c = compression stress, psi
F_t = tension stress, psi
F_y = yield stress, psi
G_c' = effective core shear modulus, psi
h = stringer, depth, in.
K = panel buckling constant
K_T = thermal constant, °F-in.
L = panel buckling constant
L_c = column length, in.
m = Ramberg-Osgood coefficient
M_u = applied bending moment, in./lb
P = applied axial load, lb/in.
P_A, P_{cc}, P_s = axial load, lb
Q = applied shear load, lb
S = core cell size, in.
t = thickness, in.
t_f = honeycomb panel facing, in.
t_c = core foil thickness, in.
T = temperature, °F
T_B = boundary-layer temperature, °F
V_x, V_y = core shear stiffness parameter
y, y_n = element distance from a given axis, in.
μ = Poisson's ratio
α = coefficient of expansion, in./°F-in.

Subscripts

b = stringer
c = core
cc = crippling
cR = critical buckling stress
cy = compression yield
f = facing
L = lower
n = generalize element number
R = effective modulus
s = secant, skin
T = tangent
u = upper, ultimate
w = web
x = component in x direction
y = component in y direction
1 = hot facing of honeycomb panel
2 = cold facing of honeycomb panel

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APPROACHING the optimization of structures with thermal gradients by a conventional manner yielded a maze of complex equations. Therefore, a new approach was sought. Since it is known that the strain from the axial load is constant across the section and that the thermal strains are dependent only on the constants, temperature, and α, the strain on any element or elements can be determined if the total strain on any one element is assumed. Thus, from several assumed strain distributions, the allowable strain equations, and the equilibrium equations, several compression panel configurations can be determined. These configurations can then be compared to determine the minimum-weight panel configuration. If bending enters the problem, an expression for the bending strain may be included by assuming a maximum bending strain on an element with a straight-line distribution between elements. Thus, assumed bending strain is iterated until bending equilibrium is achieved. Again several configurations are computed using different total strains on a given element in order to determine the minimum-weight configuration.

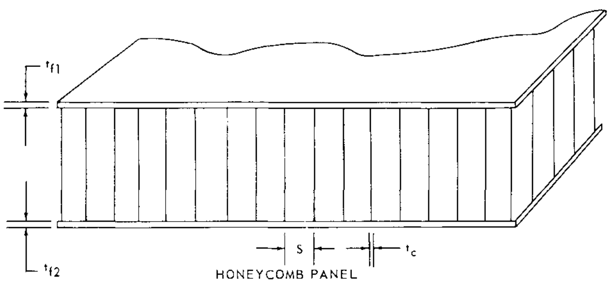

Fig. 1 Honeycomb panel.

The following sections discuss the derivation of the strain equations, equilibrium equations, allowable strain equations, and weight equations necessary to determine the optimum design. The procedures for combining these equations and developing a final minimum-weight design configuration are also discussed in detail. Examples of the successful application of these methods are presented in the paper as design curves.

Honeycomb Compression Panels

In this section we shall consider the method derived for the fixed-load honeycomb panel (Fig. 1). The design method has been derived for thermal gradients dependent on facing thickness, core density, and core depth. The facing temperatures have been expressed in an empirical tabular form as functions of the configuration parameters, so that a minimum amount of time is expended computing temperatures. However, since the final temperature and the configuration are interdependent, an iterative process is required in order to determine the final balanced design condition.

The honeycomb compression panel with a fixed load is designed to satisfy three failure modes: general panel buckling, face wrinkling, and intercell buckling. The general buckling allowable load[1] as used in the following derivation is a function of two unequal facing thicknesses with different material properties, core depth, core density, edge fixity conditions, and panel size. This equation can be written as follows:

$$P = \frac{P_M A H^2}{a^2} \tag{1}$$

$$A = \frac{\pi^2 t_{f1} t_{f2} E_1 E_2}{(1 - \mu^2)(t_{f1}E_1 + t_{f2}E_2)} \equiv \frac{\pi^2 Y}{(1 - \mu^2)} \tag{2}$$

$$H = \left(C + \frac{t_{f1} + t_{f2}}{2}\right) \equiv C + X \tag{3}$$

$$P_M = \frac{K + [(V_x/c_4) + V_y]F}{1 + L + (V_x V_y/c_4)F} \tag{4}$$

$$K = c_1 + 2c_2 + c_3 \tag{5}$$

$$F = c_1c_3 - c_2^2 + \frac{1 - \mu}{2} c_2 K \tag{6}$$

$$L = \left(c_1 + \frac{1 - \mu}{2} c_2\right) \frac{V_x}{c_4} + \left(c_3 + \frac{1 - \mu}{2} c_2\right) V_y \tag{7}$$

The core shear parameters are

$$V_y = K_1 YC/G_{cy}' \qquad V_x = K_1 YC/G_{cx}' \tag{8}$$

where

$$K_1 = \frac{\pi^2}{a^2(1 - \mu^2)} \qquad Y = \frac{t_{f1} t_{f2} E_1 E_2}{(t_{f1}E_1 + t_{f2}E_2)} \tag{9}$$

$$G_{cx}' = G_{cy}'/2 \text{ for hexcell core} \tag{10}$$

$$G_{cx}' = G_{cy}' \text{ for square cell core} \tag{11}$$

The face-wrinkling mode of failure is primarily a function of the core density and facing modulus. The allowable facing stress is expressed as follows:

$$F_{cw} = 0.5[0.066(t_c/S)^2 G_c E_c (E_s + 3E_T)]^{1/3} \tag{12}$$

It must be noted at this point, however, that the facing moduli, E_s and E_T, are functions of F_{cw}. In such a case, the Ramberg-Osgood stress-strain relation must be used. Intercell buckling is also dependent upon the Ramberg-Osgood strain relation, along with facing thickness and cell size. The allowable intercell buckling face stress is written as follows:

$$F_{ci} = 0.9E_R \left(\frac{t_f}{S}\right)^{3/2} = \frac{1.8EE_T}{E + E_T}\left(\frac{t_f}{S}\right)^{3/2} \tag{13}$$

With the allowable equations (1, 12, and 13), the design of a honeycomb compression panel can be accomplished. The requirements of the design for this derivation have included a temperature gradient, as well as the axial compression load. The applied stress under these conditions can be expressed as follows:

$$F_c \text{ applied} = -\alpha E \Delta T + (1/A_F)\Sigma \alpha E \Delta T \Delta A_F - (P/\Sigma^{\Delta A_F}) \tag{14}$$

Because of the two facings being at different temperatures and thus having different material properties and stress levels, an effective area of each element must be used. No bending relief stress is used in this analysis, since it is assumed that the panels are restrained in bending by adjacent structure. Equation (14) can now be rewritten as follows:

$$F_c \text{ applied} = -\alpha_1 E_{s1}(T_1 - T_B) + \frac{\alpha_1 E_{s1}(T_1 - T_B)t_{f1} + \alpha_2 E_{s2}(T_2 - T_B)t_{f2}}{t_{f1} + (E_{s2}/E_{s1})t_{f2}} - \frac{P}{t_{f1} + (E_{s2}/E_{s1})t_{f2}} \tag{15}$$

For the design of the minimum-weight panel, the allowable face wrinkling and general panel buckling stresses will be equal to the applied-load stress. Therefore, a relation for core density (t_c/S) can be derived by combining Eqs. (12) and (15) and solving for t_c/S:

$$\frac{t_c}{S} = \frac{11.0(F_c \text{ applied})^{3/2}}{G_c E_c (E_{s1} + 3E_{t1})^{1/2}} \tag{16}$$

Another parameter that must be determined is the core depth C. A cubic expression for C may be derived directly from the general buckling equation (1) by substituting (2–4, 8, 10, and 11) and following by algebraic manipulations to the form

$$C^3\left[FY\left(\frac{K_1^2}{G_{cx}'c_4}+\frac{K_1^2}{G_{cy}'}\right)\right] + C^2\left[K_1K + 2XFY\left(\frac{K_1^2}{G_{cx}'c_4}+\frac{K_1^2}{G_{cy}'}\right) - \frac{PK_1^2FY}{G_{cx}'G_{cy}'c_4}\right] + C\left[2XK_1K + X^2FY\left(\frac{K_1^2}{G_{cx}'c_4}+\frac{K_1^2}{G_{cy}'}\right) - K_1P\left(\frac{\{c_1+[(1-\mu)/2]c_2\}}{c_4G_{cx}'}+\frac{\{c_3+[(1-\mu)/2]c_2\}}{G_{cy}'}\right)\right] + X^2K_1K - \frac{P}{Y} = 0 \quad (17)$$

No further reduction of Eq. (17) was attempted, since the solution of the cubic will be performed on a digital computer.

All equations (13, 16, and 17) for the design of a honeycomb panel in compression have now been presented. The following design procedure has been derived to compute the minimum-weight (optimum) configuration under the stipulated conditions. The basic approach has been changed from one of stress as used in the past to one of strain. By assuming the total strain on one facing of the panel, the strain can be computed for the other facing. The strain on the second facing is expressed as follows:

$$e_2 = e_1 + \alpha(T_1 - T_2) \quad (18)$$

This equation is valid, since it has been assumed that no bending occurs in the panel.

At this point an arbitrary facing thickness t_{f1} is also assumed. With this assumed value of t_{f1} an initial estimate of the facing temperatures is computed. This initial estimate is made with a very simple equation:

$$T = (K_t/t_{f1}) + T_B \quad (19)$$

Knowing e_1 and e_2 and the temperature of the facings, the facing stress F can be computed using the Ramberg-Osgood equation in an iterative process:

$$e = (F/E)[1 + \tfrac{3}{7}(F/F_y)^{m-1}] \quad (20)$$

The yield stress F_y is obtained from material properties at temperature.

With known t_{f1} and facing stresses F_{c1} and F_{c2}, the second facing thickness t_{f2} can be computed as a function of the applied load N:

$$t_{f2} = (N - t_{f1}F_{c1})/F_{c2} \quad (21)$$

Two other parameters, t_c/S and C, remain to be determined to complete the panel configuration; they can be determined from Eqs. (15) and (17), respectively. With the final determination of panel configuration for the given load, temperature, t_{f1}, and e_1, a new estimate for temperature distribution

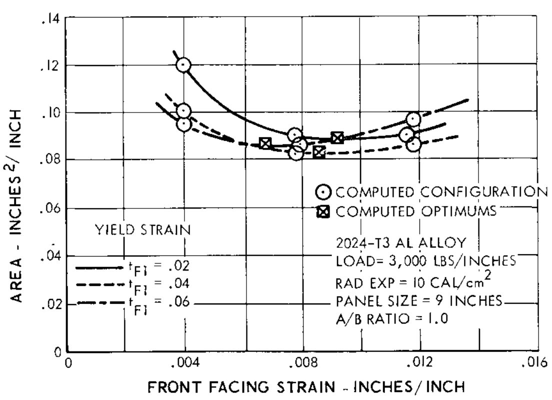

Fig. 2 Optimum strain selection.

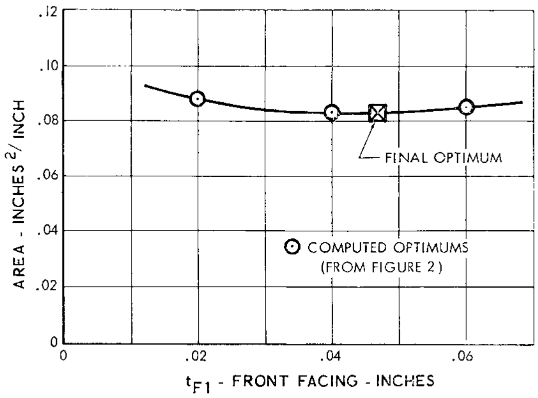

Fig. 3 Optimum t_{f1} selection.

can be made with much greater accuracy, following the method from Ref. 2.

The process of estimating the temperature and recomputing the panel configuration as described is repeated until temperature convergence is achieved. The area or relative weight of this configuration can be determined as follows:

$$A_T = t_{f1} + t_{f2} + K(t_c/S)C \quad (22)$$

where $K = 2$ for square cell, 3 for Hexcell.

With a compatible panel configuration determined for the assumed strain e_1 and facing thickness t_{f1}, another strain e_1 will be assumed. A compatible panel configuration is then determined for this assumed strain also. This process is repeated for a third time so that there are three different configurations. The strain at which the minimum weight configuration occurs (Fig. 2) can now be determined by fitting an equation for area vs e_1 through the three points, differentiating the equation and setting it equal to zero. The e_1 that satisfies $dA_T/de_1 = 0$ is the strain for the minimum weight (or area) configuration.

This whole process is repeated for two other values of t_{f1} to establish three optimum e_1 configurations as a function of t_{f1} (Fig. 2). The "absolute optimum" can now be determined by fitting an equation for the area through the three computed points as a function of t_{f1} (Fig. 3), setting the derivative (dA_T/dt_{f1}) equal to zero, solving this equation for the optimum t_{f1}, and determining the configuration compatible with the optimum t_{f1}.

Integral Skin-Stringer Panels

This section considers the optimum design of an integral skin-stringer panel with a thermal gradient under a given axial load P_s as shown in Fig. 4a. For the purpose of analysis, only one T-section will be considered (Fig. 4b). The temperatures of the skin and stringers are assumed to be functions of the skin thickness and stringer depth, respectively. It is assumed that, for an optimum design, the stringer will be stable, and the skin will be subject to local buckling.

For a given stringer spacing b_s and skin thickness t_s, the temperature of the skin T_s may be determined. In this case, the skin temperature is assumed to be related to the skin thickness by

$$T_s = (K_T/t_s) + T_B \quad (23)$$

At this point, a value of the skin strain e_s must be assumed, along with a value of the stringer depth h. The stringer temperature may now be determined by

$$T_b = (K_T/\tfrac{1}{2}h) + T_B \quad (24)$$

Since the panel is assumed to be restrained in bending, the stringer strain e_b may now be calculated by

$$e_b = e_s + \alpha(T_s - T_b) \quad (25)$$

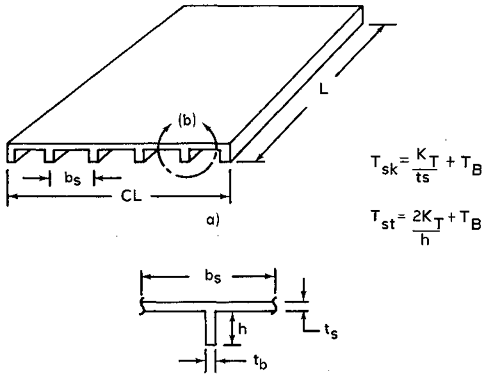

Fig. 4 Integral skin-stringer panel.

At this point, the skin stress f_s and the stringer f_b associated with the strains e_s and e_b may be calculated by the Ramberg-Osgood Eq. (20). The stringer thickness t_b may be determined by combining the buckling equation and the Ramberg-Osgood equation:

$$e_{cr_b} = e_{cb} = K_{cb}(t_b/h)^2 = F_{cyb}/0.7E_b \tag{26}$$

where e_{cy} is the compression yield strain of the stringer and K_{cb} is the buckling coefficient ($K_{cb} = 0.385$ for one edge free), and F_{cyb} is the compression yield stress of the stringer. Therefore,

$$t_b = h(F_{cyb}/0.7E_bK_{cb})^{1/2} \tag{27}$$

Since the skin is allowed to buckle, the effective area coefficient C_s must be calculated. From Sec. 7-5 for Ref. 5, the approximate value of C_s is

$$\begin{aligned} C_s &= (e_{cr_s}/e_s)^{1/2} \qquad & e_s > e_{cr_s} \\ C_s &= 1 \qquad & e_s \leq e_{cr_s} \end{aligned} \tag{28}$$

where $e_{cr_s} = K_{cs}(t_s/b_s)^2$. The axial load equilibrium equation may be written and solved for h:

$$h = \frac{P_s - f_st_sCLC_s}{f_b[(CL/b_s) + 1]t_b} \tag{29}$$

The value of h calculated is compared with the value of h originally assumed, and the process is iterated on h. The panel must now be checked to see if it is critical as a column. The critical load P_{cc} may be determined by

$$P_{cc} = \Sigma f_nA_nC_n = f_st_sCLC_s + F_{cyb}t_bh[(CL/b_s) + 1] \tag{30}$$

Note that f_b has been set equal to F_{cyb} for a column cutoff stress.

The allowable column load P_A may be determined from Sec. 14.6 of Ref. 3:

$$P_A = P_{cc}\left[1 + \frac{P_{cc}(L_c/\rho)^2}{4\pi^2E\Sigma A_n}\right] \tag{31}$$

or

$$P_A = P_{cc}\left[1 + \frac{P_{cc}L_c^2(t_sb_sC_s + t_bh)}{4\pi^2E_b[t_sCLC_s + (CL/b_s + 1)t_bh]\{\bar{y}^2t_sb_sC_s(E_s/E_b) + t_bh[(t_s + h)/2 - \bar{y}]^2 + t_bh^3/12\}}\right] \tag{32}$$

The distance to the neutral axis $\bar{y}$ may be located by

$$\bar{y} = \frac{\Sigma E_nA_ny_n}{\Sigma E_nA_n} = \frac{E_bt_bh(h + t_s)/2}{E_st_sb_s + E_bt_bh} \tag{33}$$

Next, a new value of skin strain e_s is chosen, and the entire procedure just outlined is repeated. After several values of e_s have been chosen, a plot is made of e_s vs total area A_T, where

$$A_T = t_sCL + (CL/b_s + 1)t_bh \tag{34}$$

A plot also is made of e_s vs P_A. At the point where the column's allowable load P_A is equal to the applied load P_s, the curve e_s vs A_T is cut off, because strains larger than this produce column buckling (see Fig. 5). The cutoff value of e_s is the optimum skin strain for a given skin thickness t_s and stringer spacing b_s. Next, a new value of skin thickness t_s is chosen, and the foregoing procedure is repeated. After several values of t_s have been chosen, a plot is made of t_s vs A_T (Fig. 6). The minimum of this curve gives the optimum skin thickness for a given stringer spacing b_s. Finally, several other values of stringer spacing b_s are chosen, and the steps previously outlined are repeated. A plot is made of b_s vs A_T (Fig. 7). The minimum of this curve is then the final optimum configuration.

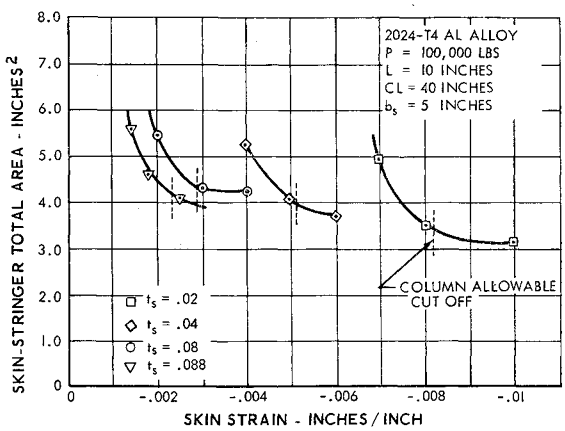

Fig. 5 Optimum strain selection.

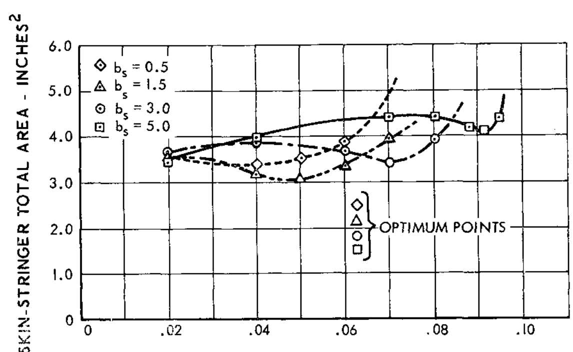

Fig. 6 Optimum t_s selection.

The allowable load for the optimum configuration was determined by the methods outlined in Ref. 4. It was found that the allowable load of the T-section checked very closely with the applied load (less than 3% difference). This close approximation between the allowable and applied load indicates the accuracy of the design method.

I-Beams

An I-beam is assumed to be loaded, as shown in Fig. 8, where M_u is an applied moment (inch-pounds) and Q is an applied shear force (pounds) and is assumed to be unrestrained in bending. The temperature of the upper skin element s is assumed to be a function of the skin thickness. The temperature of the web element T_s and the lower cap element T_L are assumed to remain constant.

According to Gatewood and Gehring,[4] the strain on any element n may be written as

$$e_n = -\alpha_nT_n + (e_{ap} + e_p + e_T) + (y_n/c)(K_{ap} + K_p + K_T) \tag{35}$$

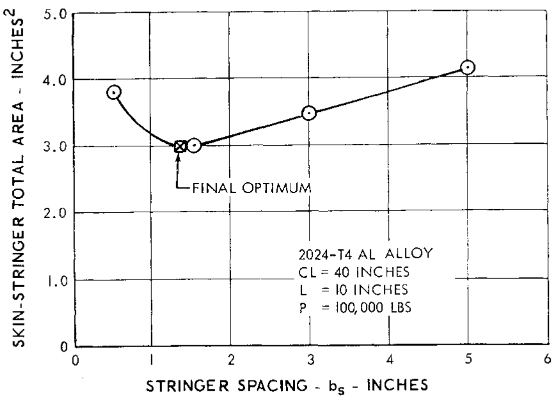

Fig. 7 Optimum stringer spacing selection.

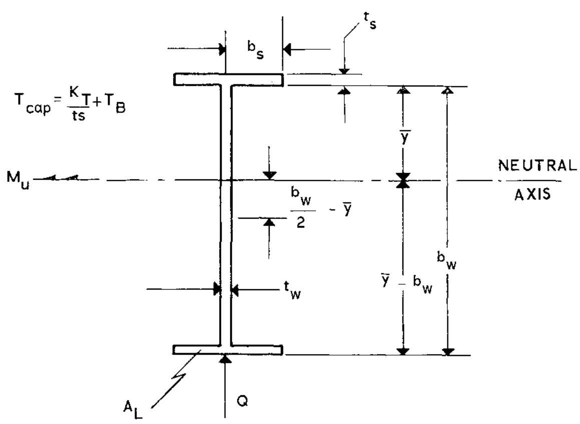

Fig. 8 *I*-beam.

where y_n is the distance from the centroid of the element n to the neutral axis, and c is the maximum value of y_n.

The strain equations of the web and lower cap may then be written in terms of e_s as

$$e_w = e_s + \alpha(T_s - T_w) + \tfrac{1}{2}e_{Lb} \qquad (36)$$

$$e_L = e_s + \alpha(T_s - T_L) + e_{Lb} \qquad (37)$$

where $e_{Lb} = (y_n/c)(K_{ap} + K_p + K_T)$ for the lower cap.

First, a value of the skin thickness t_s is assumed, and the skin temperature T_s determined from Eq. (23). Values are now assumed for both e_s and e_{Lb}. The strains e_w and e_L may now be calculated by Eqs. (36) and (37). The stresses associated with these strains f_s, f_w, and f_L, along with the secant moduli E_{ss}, E_{sw}, and E_{sL}, may be determined from the Ramberg-Osgood stress-strain relationship, Eq. (20). The skin stress f_s is assumed to have a maximum value of F_{cy}.

The web is designed to carry the average axial strain e_w and shear strain without buckling. The allowable shear stress F_{scr} can be expressed as follows:

$$F_{scr} = K_{w_s}(t_w/b_w)^2 E_{T_w} \qquad (38)$$

where K_{w_s} is the shear buckling coefficient and E_{T_w} is the tangent modulus. The allowable axial stress is expressed as

$$F_{cr} = K_w(t_w/b_w)^2 E_{T_w} \qquad (39)$$

where K_w is the compression buckling coefficient. The applied shear stress and axial stresses are

$$f_s = Q/t_w b_w \qquad (40)$$

$$f_w = e_w E_{sw} \qquad (41)$$

These values are combined in the shear and axial load combined stress equations to derive an expression for t_w as follows:

$$\pm(f_w/F_{cr})^2 + (f_s/F_{scr})^2 = 1 \qquad (42)$$

The sign of the first term of Eq. (42) depends on the stress f_s. If the web is in compression the sign is plus, and if the web is in tension the sign is minus.

Solving for t_w,

$$t_w^6 \pm (e_w b_w^2/K_w)^2 t_w^2 - (Q b_w/K_{w_s} E_{T_w})^2 = 0 \qquad (43)$$

Equation (43) can now be solved for t_w. The lower cap area A_L may be determined by

$$A_L = M_u/b_w F_{tu_L} \qquad (44)$$

where F_{tu_L} is the ultimate tension stress of the lower cap.

The axial load equilibrium equation may now be written as

$$2b_s t_s e_s E_{s_s} + e_w b_w t_w E_{sw} + e_L A_L E_{sL} = 0 \qquad (45)$$

Solving Eq. (45) for $2b_s t_s$,

$$2b_s t_s = A_s = \frac{-e_w b_w t_w E_{sw} - e_L A_L E_{sL}}{e_s E_{s_s}} \qquad (46)$$

where A_s is the area of the upper cap.

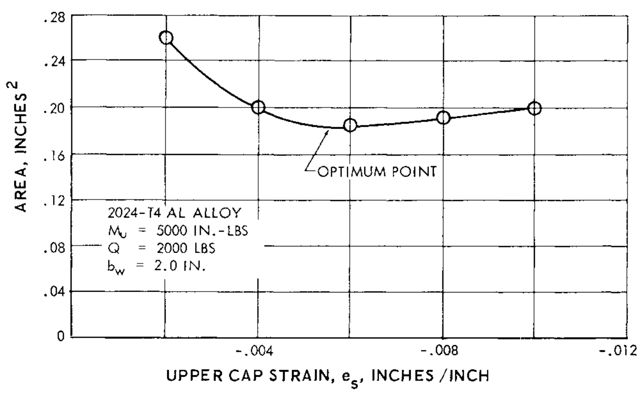

Fig. 9 Optimization curve.

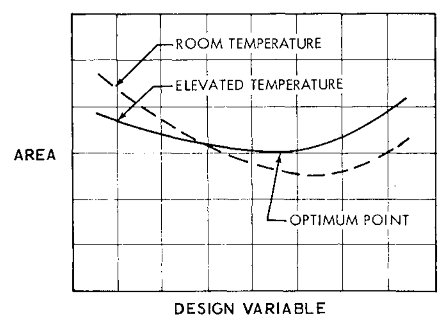

Fig. 10 Superposition of room and elevated temperature optimization curves.

Using the upper skin as a reference axis, the distance to the neutral $\bar{y}$ axis may be calculated by

$$\bar{y} = \frac{(b_w^2/2)t_w E_{sw} + b_w A_L E_{sL}}{A_s E_{s_s} + b_w t_w E_{sw} + A_L E_{sL}} \qquad (47)$$

The moment equilibrium equation may be written as

$$M_u = -e_s E_{s_s} A_s y - e_w E_{s_w} b_w t_w (y - b_w/2) - e_L e_{s_L} A_L(\bar{y} - b_w) \qquad (48)$$

Substituting Eqs. (36) and (37) into Eq. (48) for e_L and e_w and solving for e_{Lb},

$$e_{Lb} = \frac{M_u + e_s E_{s_s} A_s \bar{y} + E_{s_w} t_w b_w (\bar{y} - b_w/2)[e_s + \alpha(T_s - T_w)]}{-E_{sL} A_L(\bar{y} - b_w) - \frac{1}{2}E_{sw} t_w b_w (\bar{y} - b_w/2)} + \frac{[e_s + \alpha(T_s - T_L)]E_{sL} A_L(\bar{y} - b_w)}{-E_s A_L(\bar{y} - b_w) - \frac{1}{2}E_{sw} t_w b_w (\bar{y} - b_w/2)} \qquad (49)$$

This value of e_{Lb} is then compared with the original value of e_{Lb} assumed and the process iterated on e_{Lb}.

The upper skin is assumed to remain stable for an optimum design. Therefore,

$$e_s = -K_{cs}(t_s/b_s)^2 \qquad b_s = A_s/2t_s \tag{50}$$

or

$$t_s = (A_s/2)^{1/2}(-e_s/K_{cs})^{1/4} \tag{51}$$

This value of t_s is then compared with the original value of t_s assumed and the process iterated on t_s.

Next a new value of the skin strain e_s is chosen, and the entire procedure outlined previously is repeated. After several values of e_s are chosen, a plot is made of A_T vs e_s (Fig. 9), where A_T is the total area of the I-beam and is calculated by

$$A_T = A_s + b_w t_w + A_L \tag{52}$$

Also, a plot must be made with room temperature values to determine the critical area. The minimum of this composite curve (combination of elevated and room temperature curves) is the point of optimum design. A typical example is shown in Fig. 10.

Restrictions and Applications

Optimization methods have been developed for honeycomb panels, integral skin-stringer panels, and I-beams subjected to variable temperature gradients. However, as with all methods, certain restrictions and limitations must be imposed.

For example, the I-beam optimization method is for a variable mold line, since the beam depth b_w is the distance between centroids of the upper and lower caps. The mold line varies, depending on the skin thickness, but this variation is small, so that a good approximation may be made of the optimum design. If the design limits permit, optimum depth b_w can be found for the I-beam by extending the method previously outlined to include a variable depth b_w.

The optimum design at elevated temperatures may not be optimum at room temperature and may, in fact, not be able to sustain the applied load at room temperature. Therefore, the methods presented here must be used to determine if the structure is indeed critical at room temperature. A plot may be made of the room temperature optimum curve superimposed upon the elevated temperature optimum curve as shown in Fig. 10. The intersection of these two curves is the optimum design point when both room temperature and elevated temperature are considered, except when the minimum of one of the curves lies inside the envelope described by the two curves.

This principle of superposition of optimum curves may be extended to other parameters produced by different conditions and a design envelope established to determine an optimum structure that will satisfy all of the conditions imposed upon it.

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Low-Altitude, High-Speed Handling and Riding Qualities

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The results of a combined flight and ground-based dynamic flight simulator study of the handling and riding qualities problems associated with low-altitude, high-speed flight are presented in this paper. Wide variations of the longitudinal stability and control characteristics, which can be considered representative of current and future strike aircraft, were pilot evaluated. The influence of these stability and control characteristics, as well as the effects of low-altitude turbulence on the pilots' terrain-following performance, were measured. The results of this investigation are presented in terms of iso-opinion and iso-performance boundaries defining the desired and required combinations of stability and control parameters for low-altitude, high-speed flight. These acceptance boundaries are significantly different from the boundaries presently defined in the Military Specifications. Combinations of vehicle and control system characteristics, which tend to become unstable when coupled with the pilots' response (i.e., pilot-induced oscillations), have been defined.

Nomenclature

A	= numerator time constant in pitch acceleration-gust velocity equation, sec
D_{sp}	= $s^2 + 2\zeta\omega_n s + \omega_n^2$, 1/sec^2
D_{sp}'	= $(s^2/\omega_n^2) + (2\zeta s/\omega_n) + 1$
f_n	= natural frequency, cps
F_s	= stick force, lb
F_s/n_z	= stick force per unit load factor, lb/g
g	= acceleration due to gravity, 32.2 ft/sec^2
h	= altitude, ft
$\ddot{h}$	= climb acceleration, ft/sec^2
h_e	= altitude error, ft

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